William Kingdom Clifford (1845-1879) held a Chair in Applied Mathematics at UCL from 1871 and died of TB at the age of 33. Appointed Fellow of the Royal Society in 1874, he was an infuential mathematician both then and now, with his major contributions being in geometry, especially non-Euclidean geometry. Clifford algebras continue to be studied today and find application in, for

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who taught me as an undergraduate meant I wanted to choose a topic with which I have a particular personal interest – something which would allow me to offer my own perspective. So, having found myself increasingly involved in public engagement of mathematics over the years, I thought this would be a good opportunity to refect on my experience of promoting mathematics within science communication.

I have always been a fan of mathematics (even before I became a practitioner) and spent a great deal of my youth reading popular books on mathematics by Ian Stewart and Simon Singh, amongst others. But, as much as I enjoyed the rich history and intriguing insights that mathematics has to offer, I always felt disappointed with the way the subject is viewed by the outside world.

This disappointment has only been magnifed as I advanced through my academic career and realised just how much mathematics has to offer - and how much the general perception of the subject is at odds with the reality. Since leaving UCL mathematics and going to work in an interdisciplinary department, the starkness of this contrast has been highlighted to me even more than ever: because it is not just the outside world which struggles to see the value of mathematics – even other academics have no concept of the importance of our work.

It strikes me that mathematics is alone in this regard. Physicists, economists – even historians - don't seem to have this problem. As Edward Frenkel, Professor of Mathematics at the University of California, Berkeley, points out, the Higgs Boson, general relativity and quantum theory are regularly trotted out by the media and lapped up by the public giving a strong sense of the role of the subject in our modern world, but there are no equivalently high-level mathematical concepts in the public consciousness. And it does lead me to wonder – how have we failed to promote our subject so badly that no one has any idea what we do?

To some extent, I suppose the public's perspective is understandable. From spending

migration patterns and wonder why they were getting weird results, or government departments, with a fundamental lack of understanding of combinatorics, use excel spreadsheets to devise emergency response systems and wonder why the model was not scalable. And with more serious consequences, the world witnessed Sally Clark be wrongly convicted of murdering her two sons after a tragic misunderstanding of frighteningly basic probability and statistics, and 2008 saw the global financial crash, largely created due to a lack of understanding of the limitations in mathematical models.

Seeing these mistakes from the perspective of a mathematician does make them rather surprising, but I think it is important to ask ourselves – how

is the world supposed to know these are the kind of problems a mathematician could tackle in their sleep if they have no idea what it is that we do?

And so, I leave you with a plea. Please promote our subject. It's beautiful, it's powerful and it's important. And there is no reason why we should keep it to ourselves.

> Lecturer, The Bartlett Centre for Spatial Analysis, UCL

The word geometry usually conjures up images of the Ancient Greeks drawing triangles in the sand or doing unspeakable things with circles. But this is fat geometry, Euclidean geometry, and really we're interested in doing geometry on curved spaces. Of course, the Ancient Greeks knew this because they knew that the surface of the Earth is a sphere, and as the word "geometry" translates literally as "Earthmeasurement" we will start our discussion by contemplating the geometry of the sphere.

1.1. **Geodesics** The fundamental question, for geometers and airline pilots alike, is what is the shortest path between two points? The answer is a segment of a great circle: a circle on the surface of the sphere which divides it into two equal hemispheres. These curves are called geodesics (from the Greek meaning "Earth-dividing"). As long ago as the first century, the geometer Menelaus realised that you could use these geodesic segments instead of straight lines to do geometry and invented the subject of spherical geometry. This was radically different from Euclidean geometry: for example, in spherical geometry, the sum of angles in a triangle is always bigger than 180 degrees (Figure 1).

Of course, geodesics are not always length-minimising. If a pilot in Heathrow wanted to get to Heathrow, she wouldn't fy all the way around a geodesic till she landed back where she began. Similarly, if she wanted to fy to Lisbon she would go around the geodesic segment that passes over the English Channel, not the one passing over the Arctic circle. However, they are always locally length minimising: each short segment minimises distance between its endpoints.

We use the same name, geodesic, for a locally length minimising path in any curved space. A fundamental problem of geometry is to determine the geodesics in a given space. For example, in general relativity, the geodesics in spacetime are the trajectories of particles in free-fall in a gravitational feld. For example, the geometry of our solar system is, to some approximation, the so-called Schwarzschild geometry: analysing geodesics in Schwarzschild geometry tells you about the trajectories of planets under the infuence of the sun's gravity and allows you to predict the perihelion shift of Mercury. Indeed, when science documentaries tell you about what happens when you fall into a black hole they are paraphrasing statements about the behaviour of geodesics in Schwarzschild geometry.

Writing explicit equations for geodesics (as we can on the sphere) is usually not possible and it is actually remarkable we can say anything about them. But sometimes one can make nontrivial statements about the qualitative behaviour without solving the geodesic equations explicitly. For example, it is sometimes possible to guarantee the existence of a closed geodesic loop, like a great circle on the sphere. This is

the question I want to focus on today: when is there a closed geodesic loop in a given curved space?

On the sphere we have actually seen two different types. There is the great circle, and there is the constant loop (staying at the same point). Obviously the constant loop minimises length, while the great circle is geodesic but not length-minimising. You should think of this by analogy with critical points of functions, where the derivative vanishes: there are maxima or minima and there are turning points. Indeed, there is a function on the space of loops which assigns to each loop its length (Figure 2). The constant loop sits at a minimum of this function. The great circle sits at a turning point. Of course the space of all loops is infnite-dimensional and fnding turning points of functions on infinite-dimensional spaces is harder than the usual problem in single-variable calculus. The condition for being a turning point is that the loop solves the Euler-Lagrange equation.

You can convince yourself that this picture makes sense by thinking about elastic bands. An elastic band tries to minimise its tension, which is proportional to its length, so it will always seek out a geodesic. You can wrap an elastic band around a sphere, with diffculty, and make it sit precisely on a great circle. But if you perturb it slightly it will shrink and shrink until it pings off and hits someone in the audience. If it were bound, magnetically, to the surface of the sphere then it would just go on shrinking down to a single point. In our picture of the length function, we have pushed the elastic band off the turning point and it has fallen down the graph of the length function until it hit the constant loop at the bottom (Figure 3). It always travels down the direction of steepest descent, that is it contracts as effciently as possible. This means that evolves according to a partial differential equation called the harmonic map fow: just as the heat fow equation describes how a temperature distribution evolves in time, the harmonic map fow governs the contraction of loops to geodesics. These equations are very similar in nature: they are so-called parabolic equations, which tend to smooth out irregularities and converge exponentially fast towards an eventual steady state.

One of the major themes of modern geometry and physics is the study of critical points and gradient fows of functions on infinite-dimensional spaces.

1.2 **Geodesics from the Section Now what happens if I use a more interesting**

space? The surface of a doughnut, for example? This is called a torus and what makes it most interesting is its nontrivial topology. It has a hole in it. We can find loops which wrap around this hole, like the loop in Figure 4. It is not possible to shrink to a single point whilst staying on the surface of the torus. So suppose I concentrate on those loops which can be obtained by deforming and I restrict the length function to those loops. This new function has an interesting minimum at the point in fact it has a circle's worth of minima like

 and in Figure 4 obtained by rotating around the torus. If I perturb an elastic band wrapped around it contracts back to one of these minima via harmonic map fow.

We obtain interesting closed geodesics whenever there is a loop which cannot be contracted to a point: just fow along the harmonic map fow and you end up with a noncontractible geodesic. Of course you need some hard analysis to

also known as the *harmonic map fow.*

prove that properly but, modulo that analysis, we now have a theorem guaranteeing us the existence of closed geodesics: if there is a noncontractible loop then there is a noncontractible geodesic. Of course, there can be different kinds of noncontractible loops in a space, so let us study noncontractible loops more carefully.

1.3. **The fundamental group** Fix a point *x* in your space *X*. Consider the set of all deformation classes of loops starting and ending at x . This set is called the fundamental group $_1$ (*X*). It is called a group because you can multiply loops: to multiply loops *a* and *b* starting and ending at *x* you just concatenate them (Figure 5). The "inverse" of a loop *a* -1 is just the loop *a* run in reverse.

For example, let *X* be the unit circle in the plane: loops in the circle are determined up to deformation by the number of times they wind around. Concatenating a loop with winding number *a* and one with winding number *b* gives you a loop with winding number *a b.* A loop with winding number zero can be contracted to a point. So the fundamental group of the circle is the group of integers: concatenation corresponds to addition.

For the torus it is more interesting: there are two circles to wind around. Let us call one loop and one loop . We have the interesting phenomenon that best illustrated with a picture (Figure 6). So the fundamental group is \mathbb{Z}^1 : a loop is completely determined by two winding numbers.

Our theorem now says: for every element of the fundamental group there is a noncontractible geodesic loop representing that element¹.

1.4.

heads on the desk trying to work out what went wrong with their code.

It would be nice if we could write a computer program *A*

Three academics from the Faculty of Mathematical and Physical Sciences have been recognised for their outstanding teaching in the annual Provost's Teaching Awards.

 Andrew Fisher (UCL Physics and Astronomy) Adam Townsend (UCL Mathematics) Andrew Wills (UCL Chemistry)

Adam Townsend, a PhD student in the Department of Mathematics was given the award for being the best postgraduate teaching assistant in all of UCL this year. Adam taught the Mathematical Methods for Arts and Sciences course, part of UCL's new degree in arts and sciences.

The interdisciplinary Bachelor of Arts and Sciences (BASc) course, similar to liberal arts degrees in the US, is very different from traditional British university teaching, and it presents a number of challenges for academics. Students come from a variety of backgrounds, and with varied interests and future plans,

so crafting a course which meets all their expectations needs a lot of skill.

"In teaching a mathematical methods course for UCL's new BASc, I faced a challenge: how do you bring a traditional mathematics education to such an innovative programme?" Adam says. "Engagement using social media has been a wonderful answer, providing lively forums throughout the year as well as allowing some interesting micromanaging.

Setting up peer-learning sessions over breakfast worked out fantastically for both understanding

Rod Halburd gave his Inaugural Lecture

some number-theoretic properties of solutions

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on Wednesday 13

March 2013.

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Roughly speaking, an equation (e.g., a differential or discrete equation or a cellular automaton) is said to be integrable if it is in some sense solvable or at least if its solutions can be characterized in a particularly nice way. The integrability of an equation is not always obvious. In this talk I will describe various properties that can be used as integrability detectors. In particular, I will describe the behaviour of solutions of some differential and difference equations in the complex plane and

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successfully combined mathematical research

A new category of awards this year were the UCLU Student Choice Teaching Awards which was entirely student-led in setting the award categories and criteria, and in making the nominations and decisions as to who wins the awards. Isidoros Strouthos has won awards in 2 such categories: 'Outstanding Teaching' and 'Outstanding Personal Support'.

The overarching principle of these awards are

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Our former colleague Peter Higgs has been awarded the 2013 Nobel Prize for Physics, for his work in the 1960s that led to the concept of a mass-giving particle now known as the Higgs Boson.

Peter Higgs was a temporary lecturer in the UCL Department of Mathematics from 1958 until 1960. In 1960 he accepted a permanent appointment at the University of Edinburgh and has stayed at Edinburgh ever since.

In 2010 UCL awarded Peter Higgs an honorary doctorate of science. The photo was taken in 2010 at the dinner following the conferment of the honorary doctorate. Sitting left to right: Peter Higgs, Roger Penrose and Dima Vassiliev.

The poor quality of the photo is due to a number of reasons: a) I forgot to bring a camera so I had to use somebody's mobile phone; b) the lighting in the Jeremy Bentham Room was poor and c) in the absence of a professional photographer I had to call upon the photographic skills of the UCL Provost Malcolm Grant.

Professor of Mathematics

The Departmental Colloquium resumed its work in the academic year 2013-2014. The first speaker was Richard Melrose from MIT. On 15 October 2013 he delivered a talk entitled "Loops, spin and stringstructures".

Previous Departmental Colloquia featured talks by Frank Smith (UCL), John Toland (University of Bath), John Ball (Oxford), Brian Davies (King's College London), Jeremy Gray (UCL and Open University), Nick Trefethen (Oxford), John Ockendon (Oxford) and Artur Avila (Institut de Mathematiques de Jussieu and IMPA).

- Bosanquet Prize

- Kestelman First Year Prize

- Kestelman Second Year Prize - shared

- Andrew Rosen Second Year Prize

- Andrew Rosen Final Year Prize

- Nazir Ahmad Third Year Prize

- Stevenson Prize

- The Ellen Watson Memorial Scholarship in Applied Mathematics

- Mathematika Prize

- Diagonal resolutions for the metacyclic groups G (pq)
- **Waves on fexible surfaces**
- Shadow boundaries of convex bodies
- Stably free modules over virtually free groups
- Series representations and approximation of some quantile functions appearing in fnance
- Dynamics of gene regulatory networks in the immune system
	- Problems in convex geometry
	- Metaplectic cusp forms on the Group SL2(Qi)
	- Models of interannual mid-latitude sea surface temperature variability
		- Improved characterisation and modelling of microbubbles in biomedical

applications

- Partition problems in discrete geometry
- An approach to the congruence subgroup problem via fractional weight modular forms Linear and nonlinear free surface fows in electrohydrodynamics

Awarded in the MSc Mathematical Modelling programme for the best overall performance.

Supervisor: Professor Dmitri Vassiliev **Provisional title of thesis:**

To understand the concept of mathematical fermions, one needs to mention about spinors. The mathematical concept of spinors has been introduced by E. Cartan since 1913 and then developed by many other mathematicians

and theoretical physicists. More precisely, the language of Clifford algebra, Lie algebra, representation theory and physical description of fermions are intimately related by the concept of spinors. Moreover, the idea of spinors was encoded within the Dirac feld, which was later generalised to the concept of ferminoic felds. However, the recent research developed by Professor Dmitri Vassiliev has shown that by employing microlocal analysis to PDEs, one can naturally obtain geometric concepts such as metric, connection, torsion and spinor. The striking part of the research is that it unveils a new idea of mathematical fermions by means of microlocal analysis. Therefore, my role in the research is to apply further microlocal techniques together with spectral theory to furnish the new approach of mathematical fermions, which is supposed to be a generalisation of the old one. Apart from such a generalisation, our research also aims at deriving geometry contents by performing microlocal analysis and spectral theory.

Dr Álvaro Cartea is my personal advisor and the current research supervisor. Dr Sebastian del Baño Rollin supervised my completion of MSc degree dissertation, which has the titile of: can be generated by a load moving on the ice sheet. The research is motivated by applications to transport systems in cold regions, where bodies of water are transformed into roads and runways and where air-cushioned vehicles are used to break the ice.

My supervisor is Professor William Shaw. My research interest is as follows: Pricing in incomplete markets with jumps and, in particular, when jumps are directed towards fundamental value of the stock; asymptotic methods for pricing Asian Derivatives.

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The J J Sylvester Scholarship Fund was set up in 1997, on the centenary of the death of J J Sylvester, one of the most gifted scholars of his generation. The Fund aims to award a scholarship to help support a gifted graduate mathematician.

You can make your gift to UCL online, by telephone or by post. Donations may be made

friend and mentor Ambrose Rogers with whom he later worked as a research student. Rogers was appointed to the Astor Chair in Mathematics at UCL in 1958 and Howard came to London at that

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The Mathematics class of 1963 held a 50th anniversary reunion on 11th October 2013. The group which included a few friends from other departments were shown around by Zoe Wright and Bonita Carboo, thanks to the help of Briony McArdle, Manager of Alumni Relations.

The group met in the Print Cafe at 2pm and had a light lunch in a bustling (and to our old ears noisy) atmosphere. Zoe then guided us to the Mathematics Department which we were glad to discover was still in the same building. There we were met by Bonita who was most helpful in showing us the reading and lecture rooms. Much had changed since our day, not least because the intake is almost 5 times what it was in the 60's. We were also given a good

insight into the highly successful (and profitable) mathematics department of today by former head of department Dima Vassiliev....

We also had a look downstairs where the main Union bar used to be, and where social events used to be held. This now looked very different, so we reverted to type and trooped off to the pub. After a pleasant hour or two in the Bricklayers Arms, we had an excellent meal in Elena's Etoile in Charlotte Street. All agreed we should repeat the reunion before too many of us dropped off the perch.

Sadly I did not have contact details for most of those who were in our year, so anyone interested in joining the next reunion should contact Briony McArdle who will put you into contact with us.

Mathematics 1963

